Chapter 15
Waves

1 Marks Questions

1.Explosions on other planets are not heard on earth. Why?

Ans. This is because no material medium is present over a long distance between earth and planets and is absence of material medium for propagation, sound waves cannot travel.

2.Why longitudinal waves are called pressure waves?

Ans. Because propagation of longitudinal waves through a medium, involves changes in pressure and volume of air, when compressions and rarefactions are formed.

3.Why do tuning forks have two prongs?

Ans. The two prongs of a tuning fork set each other is resonant vibrations and help to maintain the vibrations for a longer time.

4.Velocity of sound increases on a cloudy day. Why?

Ans. Since on a cloudy day, the air is wet i.e. it contains a lot of moisture, As a result of which the density of air is less and since velocity is inversely proportioned to density, hence velocity increases.

5.Sound of maximum intensity is heard successively at an interval of 0.2 second on sounding two tuning fork to gather. What is the difference of frequencies of two tuning forks?

Ans. The beat period is 0.2 second so that the beat frequency is fb = $\frac{1}{0.2}$ = 5HZ. Therefore, the difference of frequencies of the two tuning forks is 5HZ.

6.If two sound waves has a phase difference of 60^0, then find out the path difference between the two waves?

Ans. Phase difference, $\Phi = 60^0 = \frac{\pi}{3}$ rad

Now, in general for any phase difference, (Φ), the path difference (x) :→

$$\phi = \frac{2\pi}{\pi}x$$

$$Given\ \phi = \frac{\pi}{3}, x = ?$$

$$\frac{\pi}{3} = \frac{2\pi}{\pi} \times x$$

$$x = \frac{\pi}{3} \div \frac{2\pi}{\pi}$$

$$x = \frac{\not{\pi}}{3} \times \frac{\pi}{2\not{\pi}}$$

$$x = \frac{\pi m}{6}$$

7.If the displacement of two waves at a point is given by:-

$$Y_1 = a \sin wt$$

$$Y_2 = a \sin\left(wt + \frac{\pi}{2}\right)$$

Calculate the resultant amplitude?

Ans . If a_1 = amplitude of first wave

a_2 = amplitude of second wave

a_r = resultant amplitude

ϕ = phase difference between 2 waves

then $a_r = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$

In our case $a_1 = a$; $a_2 = a$; $\phi = \frac{\pi}{2}$ so

$$a_r = \sqrt{a^2 + a^2 + 2a \times a \operatorname{Cos}\left(\frac{\pi}{2}\right)}$$

$$= \sqrt{2a^2} \qquad \left(\because \operatorname{Cos}\left(\frac{\pi}{2}\right) = 0\right)$$

$$a_r = \sqrt{2}\,a$$

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8. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Ans. Speed of sound in the tissue, v = 1.7 km/s = $1.7 \times 10^3\ m/s$

Operating frequency of the scanner, v = 4.2 MHz = $4.2 \times 10^6\ Hz$

The wavelength of sound in the tissue is given as:

$$\lambda = \frac{v}{V}$$

$$= \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4}\, m$$

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9. Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a traveling wave, (ii) a stationary wave or (iii) none at all:

(a) $y = 2\cos(3x)\sin(10t)$

(b) $y = 2\sqrt{x - vt}$

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(d) $y = \cos x \sin t + \cos 2x \sin 2t$

Ans.(a) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation.

(b) The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave.

(c) The given equation represents a travelling wave as the harmonic terms kx and ωt are in the combination of $kx - \omega t$.

(d) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves.

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10. A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to $\frac{1}{20}$ or 0.05 Hz?

Ans.(a) (i)No

(ii)No

(iii)Yes

(b) No

Explanation:

(a) The narrow sound pulse does not have a fixed wavelength or frequency. However, the speed of the sound pulse remains the same, which is equal to the speed of sound in that medium.

(b) The short pip produced after every 20 s does not mean that the frequency of the whistle is $\frac{1}{20}$ or 0.05 Hz. It means that 0.05 Hz is the frequency of the repetition of the pip of the whistle.

2 Marks Questions

1.A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this source be in resonance with the pipe if both the ends are open?

Ans. Length of pipe = L = 20cm = 0.2m

Frequency of n^{th} node = υ_n = 430 Hz

Velocity of sound = v = 340m|s

Now, υ_n of closed pipe is :→

$$\upsilon_n = \frac{(2n-1)v}{4L}$$

$$430 = \frac{(2n-1)\times 340}{4\times 0.2}$$

$$2n-1 = \frac{430\times 4\times 0.2}{340}$$

$$2n-1 = 1.02$$

$$2n = 1.02+1$$

$$2n = 2.02$$

$$n = 1.01$$

Hence, it will be the first normal mode of vibration, In a pipe, open at both ends we, have

$$\upsilon_n = \frac{n\times v}{2L} = \frac{n\times 340}{2\times 0.2} = 430 \qquad So,\ 430 = \frac{n\times 340}{2\times 0.2}$$

$$n = \frac{430 \times 2 \times 0.2}{340} \qquad n = 0.5$$

As n has to be an integer, open organ pipe cannot be in resonance with the source.

2.Can beats be produced in two light sources of nearly equal frequencies?

Ans . No, because the emission of light is a random and rapid phenomenon and instead of beats we get uniform intensity.

3.A person deep inside water cannot hear sound waves produces in air. Why?

Ans . Because as speed of sound in water is roughly four times the sound in air, hence refractive

index u = $\frac{Sin\ i}{Sin\ r} = \frac{Va}{Vw} = \frac{1}{4} = 0.25$

For, refraction $r_{max} = 90^0$, $i_{max} = 14^0$. Since $i_{max} \neq r_{max}$ hence, sounds gets reflected in air only and person deep inside the water cannot hear the sound.

4.If the splash is heard 4.23 seconds after a stone is dropped into a well. 78.4 metes deep, find the velocity of sound in air?

Ans . Here, depth of well = S = 78.4m

Total time after which splash is heard = 4.23s

If t_1 = time taken by store to hit the water surface is the well

t_2 = time taken be splash of sound to reach the top of well.

then $t_1 + t_2 = 4.23$ sec.

Now, for downward journey of stone;

u = O, a = 9.8 m|s^2, S = 78.4m,

t = t1 = ?

As, $s = ut + \frac{1}{2} at^2$

$$\therefore 78.4 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$$

$$78.4 = 4.9t_1^2$$

$$t_1^2 = \frac{78.4}{4.9}$$

$$t_1^2 = 16$$

$$t_1 = \sqrt{16} = 4\sec$$

$$Now, t1 + t2 = 4.23$$

$$4 + t_2 = 4.23$$

$$t_2 = 4.23 - 4.00$$

$$t_2 = 0.23s$$

If V = velocity of sound in air,

$$V = \frac{dis\tan ce(s)}{time(t)} = \frac{78.4}{0.23} = 340.87m|s.$$

5.How roar of a lion can be differentiated from bucking of a mosquito?

Ans. Roaring of a lion produces a sound of low pitch and high intensity whereas buzzing of mosquitoes produces a sound of high pitch and low intensity and hence the two sounds can be differentiated.

6.The length of a sonometer wire between two fixed ends is 110cm. Where the two bridges should be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio of 1:2:3?

Ans .Let l_1, l_2 and l_3 be the length of the three parts of the wire and f_1, f_2 and f_3 be their respective frequencies.

Since T and m are fixed quantities, and 2 are constant

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$f = \alpha \frac{1}{l}$$

$$or\ f_1 = \text{constant}$$

So, $f_1\ l_1$ = Constant →(1)

$f_2\ l_2$ = Constant →(2)

$f_3\ l_3$ = Constant →(3)

Equating equation 1), 2) & 3)

$f_1\ l_1 = f_2\ l_2 = f_3\ l_3$

Now, $l_2 = \frac{f_1}{f_2} l_1$

$$l_2 = \frac{1}{2} l_1 \rightarrow (4) \left(\frac{f_1}{f_2} = \frac{1}{2} \right) Given$$

Also, $l_3 = \frac{f_1}{f_3} l_1$

$$l_3 = \frac{1}{3} l_1 \left(\frac{f_1}{f_3} = \frac{1}{3} (given) \right)$$

Now, Total length = 110cm

i.e $l_1 + l_2 + l_3 = 110cm$

$$l_1 + \frac{1}{2} l_1 + \frac{1}{3} l_1 = 110$$

$$\frac{11 l_1}{6} = 110$$

$$l_1 = \frac{\not{1}\not{1}\not{0}^{10} \times 6}{\not{1}\not{1}} = 60cm$$

i. e. $l_1 = 60cm$

$$l_2 = \frac{l_1}{2}$$

$$l_2 = 30cm$$

$$l_2 = \frac{60}{2}$$

Now,

$$l_3 = \frac{l_1}{3} \qquad l_3 = \frac{60}{3}$$

$$l_3 = 20cm$$

7.If string wires of same material of length l and 2l vibrate with frequencies 100HZ and 150 HZ. Find the ratio of their frequencies?

Ans. Since frequency = f of a vibrating string of mass and Tension = T is given by:→ l = length

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Let for first case, f_1 = 100HZ ; l_1 = l ; T_1 = Initial Tension

For second case, f_2 = 150HZ ; l_2 = 2l ; T_2 = Final Tension

So, $f_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$

$$f_1 = \frac{1}{2l}\sqrt{\frac{T_1}{m}}$$

$$100 = \frac{1}{2l}\sqrt{\frac{T_1}{m}} \rightarrow (1)$$

$$and\ f_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$150 = \frac{1}{2_1 l}\sqrt{\frac{T_2}{m}} \rightarrow (2)$$

Divide equation 1) by equation 2)

$$\frac{100}{150} = \frac{\frac{1}{2l}\sqrt{\frac{T_1}{m}}}{\frac{1}{4l}\sqrt{\frac{T2}{m}}}$$

$$\frac{10\cancel{0}}{15\cancel{0}} = \frac{1}{{}_1\cancel{2}\cancel{l}}\sqrt{\frac{T1\times \cancel{m}\times \cancel{4}^{2}\cancel{l}}{\cancel{m}\times T_2}}$$

$$\frac{\cancel{10}\ 2}{\cancel{15}\ 3} = 2\sqrt{\frac{T_1}{T_2}}$$

$$\frac{2}{3} = 2\sqrt{\frac{T_1}{T_2}}$$

$$\frac{\not{2}}{3\times\not{2}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{1}{3} = \sqrt{\frac{T_1}{T_2}} \rightarrow \textit{Squaring}\text{ both sides}$$

$$\text{or } \frac{1}{9} = \frac{T_1}{T_2}$$

Hence, the ratio of tensions is

1 : 9

8.Two similar sonometer wires of the same material produces 2 beats per second. The length of one is 50cm and that of the other is 50.1 cm. Calculate the frequencies of two wives?

Ans. The frequency (f) of a son meter wire of length = l, mass = m and lesion = T is given by

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$Let\ \mathrm{k} = \frac{1}{2}\sqrt{\frac{T}{m}}$$

$$so,\ f = \frac{k}{l}$$

$$\text{In first case; } \mathrm{f}_1 = \frac{k}{l_1} \rightarrow 1)\ \ and\ \ f2 = \frac{k}{l_2} \rightarrow 2)$$

Subtract equation 1) & 2)

$$f_1\text{-}f_2 = \frac{k}{l_1} - \frac{k}{l_2} = k\left(\frac{1}{l_1} - \frac{1}{l_2}\right)$$

Now, given $l_1 = 50\text{cm}; l_2 = 50.1\text{cm}$

$f_1\text{-}f_2 = 2$

$$\text{so, } 2 = k\left[\frac{1}{50} - \frac{1}{50.1}\right]$$

$$2 = k\left[\frac{50.1 - 50}{50 \times 50.1}\right]$$

$$2 = K\left[\frac{0.1}{2505.0}\right]$$

$$\frac{2 \times 2505}{0.1} = k$$

$$\frac{5010 \times 10}{01.1} = k$$

50100 = k

$$so, f_1 = \frac{k}{l_1} = \frac{5010\cancel{0}}{5\cancel{0}} = 1002HZ$$

$$f_2 = \frac{k}{l_2} = \frac{\cancel{50100}^{100} \times 10}{\cancel{501}\ 1} = 1002HZ$$

9.Why are all stringed instruments provided with hollow boxes?

Ans . The stringed instruments are provided with a hollow box called sound box. When the strings are set into vibration, forced vibrations are produced in the sound box. Since sound box has a large area, it sets a large volume of air into vibration. This produces a loud sound of the same frequency of that of the string.

10.Two waves have equations:-

$X_1 = a \text{ Sin } (wt + \phi_1)$ $X_2 = a \text{ Sin } (wt + \phi_2)$

If in the resultant wave, the amplitude remains equal to the amplitude of the super posing waves. Calculate the phase difference between X_1 and X_2?

Ans. Given, the first wave:→ X_1 = a Sin $(wt + \phi_1)$

The second : $\rightarrow X_2 = a \text{ Sin } (wt + \phi_2)$ wave

Where X_1 = wave function first wave

X_2 = wave function in second wave

a = amplitude

w = Angular frequency

t = time

ϕ_1 = phase difference of first wave

ϕ_2= phase difference of second wave.

Let The resultant amplitude = 'a' and phase difference

$= \phi$ so,

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a2Cos\phi}$

a_1 = amplitude of first wave

a_2 = amplitude of second wave

ϕ = phase difference between two waves.

Now, in our case,

$$a1 = a$$

$$a2 = a$$

$$so, a = \sqrt{a^2 + a^2 + 2a \times a Cos\phi}$$

$$a = \sqrt{2a^2 + 2a^2 Cos\phi}$$

$$a = \sqrt{2a^2(1 + Cos\phi)}$$

$$a = \sqrt{2a^2 \times 2Cos\frac{\phi}{2}} \qquad \left(\because 1 + Cos\theta = 2Cos\frac{\theta}{2}\right)$$

$$a = \sqrt{4a^2 Cos\frac{\phi}{2}}$$

$$a = 2aCos\frac{\theta}{2}$$

$$\not{a} = 2\not{a}Cos\frac{\phi}{2}$$

$$\frac{1}{2} = Cos\frac{\phi}{2}$$

$$\frac{\Phi}{2} = Cos^{-1}\left(\frac{1}{2}\right) = \not{2} \times 6\not{0}^0 \; \frac{\phi}{2} = 60^0$$

$$\phi = 2 \times 60^0$$

$$\Phi = 120^0$$

So, the phase difference between X_1 and X_2 is 120^0.

11.A Tuning fork of frequency 300HZ resonates with an air column closed at one end at 27^0C. How many beats will be heard in the vibration of the fork and the air column at

0^0C?

Ans . According to the question,

Frequency of air column at 27^0C is 300 HZ

Let l = length of air column and speed of sound = V_{27}

For a pipe; closed at one end, the frequency of n^{th} harmonic is:→

$$fn = n\left(\frac{v}{4l}\right)$$

n = 1, 3, 5, 7 --------

l = length of air column

v = velocity

So, Let at 0^0C, the speed of sound = v_o then,

$$f_1 = n_1\left(\frac{v_1}{4l}\right) and\ f_2 = n_2\left(\frac{v_2}{4l}\right)$$

f_1 = frequency at 27^0C = 300HZ

f_2 = frequency at 0^0C = ?

$$\text{So, } f_1 = \frac{V_{27}}{4l}$$

$$300 = \frac{V_{27}}{4l} \rightarrow 1)$$

$$f_2 = \frac{Vo}{4l} \rightarrow 2)$$

$$Now, \frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{V_{27}}{Vo} \sqrt{\frac{(27+273)k}{(273)k}}$$

$$\frac{V_{27}}{Vo} = \sqrt{\frac{300}{273}}$$

$$\frac{V_{27}}{Vo} = \cancel{0.954}\sqrt{\frac{300}{273}}$$

$$or \frac{Vo}{V_{27}} = \sqrt{\frac{273}{300}}$$

$$\frac{Vo}{V_{27}} = 0.954$$

Vo = (0.954) V_{27}

∴ frequency of air column at 0^0C is :→

$$f_2 = \frac{Vo}{4l}(equation\ 2)$$

Using Vo = (0.954) v_{27} We get

$$f_2 = \frac{(0.954)V_{27}}{4l} \quad Now, \frac{V_{27}}{4l} = 330(equation\ \ A)$$

$$f_2 = 0.954 \times 330 = 286HZ$$

The frequency of tuning fork remains 300HZ Number of beats = $f_1 - f_2$

= 300-286 = 14 Per second

12.A vehicle with horn of frequency 'n' is moving with a velocity of 30m|s in a direction perpendicular to the straight line joining the observer and the vehicle. If the observer perceives the sound to have a frequency of n+n_1. Calculate n_1?

Ans. By Doppler effect, the apparent change in frequency of wave due to the relative motion between the source of waves and observer.

If v_2 = velocity of the listener

v_s = velocity of the source

v = velocity of sound

γ = frequency of sound reaching from the source to the listener.

γ^1 = Apparent frequency (i.e. Changed frequency due to movement of source and listener)

$$\gamma^1 = \left(\frac{V - V_2}{V - V_s}\right) \times \gamma$$

But in our case, the source and observer move at right angles to each other. The Doppler Effect is not observed when the source of the sound and the observer are moving at right argyles to each other.

So, if n = original frequency of sound the observer will perceive the sound with a frequency of n (because of no Doppler effect). Hence the n_1 = charge frequency = 0.

13.We cannot hear echo in a room. Explain?

Ans. We know that, the basic condition for an echo to be heard is that the obstacle should be rigid and of large size. Also the obstacle should be at least at a distance of 17m from the source. Since the length of the room is generally less than 17m, the conditions for the production of Echo are not satisfied. Hence no echo is heard in a room.

14.Why do the stages of large auditoriums gave curved backs?

Ans. The stages of large auditorium have curved backs because when speaker stands at or near the focus of curved surface his voice is rendered parallel after reflection from the concave or parabolic seer face. Hence the voice can be heard at larger distances.

15.Show that Doppler effect in sound is asymmetric?

Ans. The apparent frequency of sound when source is approaching the stationary listener (with velocity v^1) is not the same as the apparent frequency of sound when the listener is approaching the stationary source with a velocity v^1. This shows that Doppler Effect in sound is asymmetric.

Apparent frequency = $f^1 = \frac{V}{V - us} \times f$

This is when source approaches station any listener

V = Velocity of sound in air

u_s = Velocity of sound source = V_1

f^1 = Apparent frequency

f = Original frequency of sound

Apparent frequency $f'' = \frac{V + uo}{V} \times f$

→ This is when listener approaching stationary source $f'' = \frac{V + V^1}{V} \times f$

Since $f' = f''$ (Hence Doppler effect is asymmetric)

16.An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at both the ends vibrating in its third overtone are in resonance with a given

tuning fork. Find the ratio of length of P_1 and P_2?

Ans. Length of pipe closed at one end for first overtone, $l_1 = \frac{3\pi}{4}$

Length of pipe closed at both ends for third overtone; $l_2 = \frac{3\pi}{4}$

π= wavelength

$$l_2 = \frac{4\pi}{2} = 2\pi \qquad \therefore \frac{l_1}{l_2} = \frac{\frac{3\pi}{4}}{2\pi} = \frac{3\pi}{4\times 2\pi} = \frac{3}{8}$$

$$l_1 : l_2 = 3:8$$

17.A simple Romanic wave has the equation

Y = 0.30 Sin (314 t – 1.57x)

t = sec, x = meters, y = cm. Find the frequency and wavelength of this wave.

Another wave has the equation-

Y_1 = 0.1 Sin (314 t – 1.57x + 1.57)

Deduce the phase difference and ratio of intensities of the above two waves?

Ans.If y is in meters, then equation becomes:→

$$y = \frac{0.30}{100} Sin(314\ t\text{-}1.57x) \rightarrow (1)$$

The standard equation of plane progressive wave is

$$y = a\ \text{Sin}\ (wt - k\ x) \quad \rightarrow (2)$$

Comparing equation 1) & 2)

$w = 314, k1.57;$ $a = \frac{0.30}{100} = 3\times10^{-3} m$

$\therefore$ frequency $= f = \frac{w}{2\pi} = \frac{314}{2\pi} = 50HZ$

wave velocity, $v = \frac{w}{k} = \frac{314}{1.57} = 200 m/s$

$\therefore wavelenght = \pi = r\frac{v}{f} = \frac{200}{50} = 4m$

On inspection of the equations of the given two waves,

Phase difference,

$\Delta\phi = 1.57$ rad

$= 1.57\times\left(\frac{180}{\pi}\right) = 90^{0}$

Ratio of amplitudes of two waves $= \frac{a_1}{a_2} = \frac{0.3}{0.1} = 3$

$\therefore$ Ratio of intensities $= \frac{I_1}{I_2} = \frac{a_1^{\ 2}}{a_2^{\ 2}} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

18.The component waves producing a stationary wave have amplitude, Frequency and velocity of 8 cm, 30HZ and 180 cm/s respected. Write the equation of the stationary wave?

Ans. Since the ware equation of a travelling wave:- $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\pi}\right)$

a = Amplitude

t = time

T = Time Period

x = Path difference

π = wavelength

Let $y_1 = a \text{ Sin } 2\pi\left(\frac{t}{T} - \frac{x}{\pi}\right)$

$Y_2 = a \text{ Sin } 2\pi\left(\frac{t}{T} + \frac{x}{\pi}\right)$

(∵ It is travelling in opposite direction)

By principle of superposition, wave equation for the resultant wave = $y = y_1 + y_2$

$$y = a \text{ Sin } 2\pi\left(\frac{t}{T} - \frac{x}{\pi}\right) + a \text{ Sin } 2\pi\left(\frac{t}{T} + \frac{x}{\pi}\right)$$

$$= a\left(\text{Sin } 2\pi\left(\frac{t}{T} - \frac{x}{\pi}\right) + a \text{ Sin } 2\pi\left(\frac{t}{T} + \frac{x}{\pi}\right)\right)$$

$$\text{Using Sin C+ Sin D} = \text{a Cos } \frac{C-D}{z} . \frac{Sin\ C+D}{z}$$

$$y = 2a \text{ Cos } \frac{2\pi x}{\pi} . \frac{Sin\ 2\pi t}{T}$$

Here a = 8cm; f = 30Hz, V = 180 cm/s

$$T = \frac{1}{30} \text{ sec, } \pi\text{=wavelength} = \frac{v}{f} = VT$$

$$\pi = 180 \times \frac{1}{30} = 6cm$$

$$y = 2a \text{ Cos } \frac{2\pi x}{\pi} Sin \frac{2\pi t}{T}$$

$$y = 2\times 8\frac{\cos \cancel{2}\pi\times x}{\cancel{6}\;3}\cdot\frac{\sin \cancel{2}\pi t}{\pi\,\cancel{30}\;15}$$

$$y = 16\,\frac{\cos \pi x}{3}\cdot\frac{\sin 60\pi t}{15}$$

19.A wine of density a g/cm^3 is stretched between two clamps 1 m apart while subjected to an extension of 0.05 cm. What is the lowest frequency of transverse vibration in the wire? Let young's Modulus = y = 9$\times 10^{10}$ N/m^2 **?**

Ans. The lowest frequency of transverse vibrations is given by:→

Area = A

Density = P

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Here m = mass Per unit length = area $\times$ Density

because Density = $\dfrac{Mass}{Volume}$

m = $A\times P$

F=35.3 vib/sec

20.Give two cases in which there is no Doppler effect in sound?

Ans. The following are the two cases in which there is no Doppler effect in sound (i.e no change in frequency):-

1) When the source of sound as well as the listener moves in the same direction with the same speed.

2) When one of source | listener is at the centre of the circle and the other is moving on the

circle with uniform speed.

21. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Ans. Mass of the string, M = 2.50 kg

Tension in the string, T = 200 N

Length of the string, l = 20.0 m

Mass per unit length, $\mu = \frac{M}{l} = \frac{2.50}{20} = 0.125\,kg\,m^{-1}$

The velocity (v) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40\,m/s$$

∴Time taken by the disturbance to reach the other end, $t = \frac{l}{v} = \frac{20}{40} = 0.50\,s$

22. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20 °C = 343 m s^{-1} ?

Ans. Length of the steel wire, l = 12 m

Mass of the steel wire, m = 2.10 kg

Velocity of the transverse wave, v = 343 m/s

Mass per unit length, $\mu = \frac{m}{l} = \frac{2.10}{12} = 0.175\ kg\ m^{-1}$

For tension *T*, velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$\therefore T = v^2 \mu$

= $(343)^2 \times$ 0.175 = 20588.575 $\approx 2.06 \times 10^4$ N

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23. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s-1 and in water 1486 m s^{-1} .

Ans. (a) Frequency of the ultrasonic sound, v = 1000 kHz = 106 Hz

Speed of sound in air, v_a = 340 m/s

The wavelength (λ_r) of the reflected sound is given by the relation:

$$\lambda_r = \frac{v}{V}$$

$$= \frac{340}{10^6} = 3.4 \times 10^{-4} m$$

(b) Frequency of the ultrasonic sound, v = 1000 kHz = 106 Hz

Speed of sound in water, *vw* = 1486 m/s

The wavelength of the transmitted sound is given as:

$$\lambda_t = \frac{1486}{10^6} = 1.49 \times 10^{-3} m$$

---

24. (i) For the wave on a string described in Exercise 15.11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?

Ans. (i)

(a) Yes, except at the nodes

(b) Yes, except at the nodes

(c) No

(ii) 0.042 m

Explanation:

(i)

(a) All the points on the string oscillate with the same frequency, except at the nodes which have zero frequency.

(b) All the points in any vibrating loop have the same phase, except at the nodes.

(c) All the points in any vibrating loop have different amplitudes of vibration.

(ii) The given equation is:

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

For x = 0.375 m and t = 0

Amplitude = Displacement = $0.06\sin\left(\frac{2\pi}{3}x\right)\cos 0$

$$= 0.06\sin\left(\frac{2\pi}{3}0.375\right)\times 1$$

$$= 0.06\sin(0.25\pi) = 0.06\sin\left(\frac{\pi}{4}\right)$$

$$= 0.06 \times \frac{1}{\sqrt{2}} = 0.042\, m$$

25. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m-1. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

Ans.(a) Mass of the wire, m = 3.5×10^{-2} kg

Linear mass density, $\mu = \frac{m}{l} = 4.0 \times 10^{-2}\, kg\, m^{-1}$

Frequency of vibration, ÃŽÂ½ = 45 Hz

∴Length of the wire, $l = \frac{m}{\mu} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875\, m$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:

$$\lambda = \frac{2l}{n}$$

Where, n = Number of nodes in the wire

For fundamental node, n = 1:

λ = 2l

λ = 2 × 0.875 = 1.75 m

The speed of the transverse wave in the string is given as:

v = vλ 45 $\times$ 1.75 = 78.75 m/s

(b) The tension produced in the string is given by the relation:

$T = v_2 \mu$

$= (78.75)^2 \times 4.0 \times 10^{-2} = 248.06\ N$

---

26. Two sitar strings A and B playing the note '*Ga*' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

Ans. Frequency of string A, f_A = 324 Hz

Frequency of string B = f_B

Beat's frequency, n = 6 Hz

Beat's frequency is given as:

$n = | f_A \pm f_B |$

$6 = 324 \pm f_B$

$f_B = 330 Hz \ or\ 318 Hz$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$v \propto \sqrt{T}$

Hence, the beat frequency cannot be 330 Hz

$\therefore f_B = 318 Hz$

3 Marks Questions

1.Explain briefly the analytical method of formation of beats?

Ans. Let us consider two wares trains of equal amplitude ‘a’ and with different frequencies υ_1 and υ_2 in same direction.

Let displacements are y_1 and y_2 in time ‘t’.

w_1 = Angular vel. of first ware

$w_1 = 2\pi\upsilon_1$

$$y_1 = a \, Sin \, w_1 \, t$$

$$y_1 = a \, Sin \, 2\pi\upsilon_1 \, t$$

$$y_2 = a \, Sin \, 2\pi\upsilon_2 \, t$$

Acc. to superposition principle, the resultant displacement ‘y’ at the same time ‘t’ is:-

y = y1 + y2

$$y = a \left[Sin \, 2\pi\upsilon_1 t + Sin \, 2\pi\upsilon_2 t \right]$$

Using Sin C + Sin D = 2 Cos $\frac{C-D}{2}$ Sin $\frac{C+D}{2}$

We get,

$$y = 2a \cos \frac{2\pi(\upsilon_1 - \upsilon_2)t}{2} Sin \frac{2\pi(\upsilon_1 + \upsilon_2)t}{2}$$

$$y = 2a \, Cos \, \pi \, (\upsilon_1 - \upsilon_2)t \; Sin \, \pi(\upsilon_1 + \upsilon_2)t$$

$y = A \text{ Sin } \pi(\upsilon_1 + \upsilon_2)t$

Where, A = 2a Cos $\pi(\upsilon_1 - \upsilon_2)t.$

Now, amplitude A is maximum when,

$Cos\pi(\upsilon_1 - \upsilon_2) + = \text{ma x} + \pm 1 = \text{Cos k}\pi$

$\pi(\upsilon_1 - \upsilon_2)t = k\pi, \text{ k=0,1,2--}$

$$t = \frac{k}{(\upsilon_1 - \upsilon_2)}$$

i.e resultant intensity of sound will be maximum at times,

$$t = 0, \frac{1}{(\upsilon_1 - \upsilon_2)}, \frac{3}{(\upsilon_1 - \upsilon_2)} -----$$

Time interval between 2 successive Maximaxs = $\frac{1}{(\upsilon_1 - \upsilon_2)} - 0 = \frac{1}{(\upsilon_1 - \upsilon_2)} \rightarrow (1)$

Similarly, A will be minimum,

$\text{Cos } \pi \ (\upsilon_1 - \upsilon_2)t = 0$

$Cos \ \pi \ (\upsilon_1 - \upsilon_2)t = Cos(2k+1)\frac{\pi}{2}, k = 0,1,2,3 -$

$\pi(\upsilon_1 - \upsilon_2)t = (2k+1)\frac{\pi}{2}$

$$t = \frac{(2k+1)}{2(\upsilon_1 - \upsilon_2)}$$

i.e. resultant intensity of sound will be minimum at times

$$t = \frac{1}{2(\upsilon_1 - \upsilon_2)}, \frac{3}{2(\upsilon_1 - \upsilon_2)}, \frac{5}{2(\upsilon_1 - \upsilon_2)}$$

Hence time interval between 2 successive minim as are

$$= \frac{3}{2(\upsilon_1 - \upsilon_2)} - \frac{1}{2(\upsilon_1 - \upsilon_2)} = \frac{1}{(\upsilon_1 - \upsilon_2)} \rightarrow (2)$$

Combining 1) & 2) frequency of beats = $(\upsilon_1 - \upsilon_2)$

$\therefore \frac{No.\ of\ beats}{seconds} =$ Difference in frequencies of two sources of sound.

2.Show that the frequency of nth harmonic mode in a vibrating string which is closed at both the end is 'n' times the frequency of the first harmonic mode?

Ans . When a sting under tension is set into vibration, transverse harmonic wares propagate along its length when length of sting is fixed, reflected waves will also exist. The incident and reflected waves will superimpose on each short to produce transverse stationary waves in sting. Let a harmonic wave be set up in a sting of length = λ which is fixed at 2 ends: → x = 0 and x = L

Let the incident wave travels from left to right direction, the wave equation is:→

$$y_1 = r \text{ Sin } \frac{2\pi}{\pi}(Vt + x) \rightarrow 1)$$

The wave equation of reflected wave, will have the same amplitude, wavelength, velocity, time but the only difference between the incident and reflected waves will be in their direction of propagation So, :→

$$y_2 = r \text{ Sin } \frac{2\pi}{\pi}(vt - x)$$

$\because$ Reflected wave tracels from rght to left

Reflection, the wave will suffer a phase change of π. So,

$$y2 = r \ \text{Sin} \ \frac{2\pi}{\pi}[vt - x + \pi]$$

$$y2 = -r \ \text{Sin} \ \frac{2\pi}{\pi}[vt - x] \rightarrow (2)$$

According to the principle of superposition, the wave equation of resultant stationary wave will be:-

$$y = y_1 + y_2$$

Using equation 1) & 2)

$$y = r \ \text{Sin} \ \frac{2\pi}{\lambda}[vt + x] - r Sin \frac{2\pi}{\lambda}(vt - x)$$

$$= r\left[Sin \frac{2\pi}{\lambda}(vt + x) - Sin \frac{2\pi}{\lambda}(vt - x) \right]$$

Using Sin C - Sin D = 2 Cos $\frac{C+D}{Z} Sin \frac{C-D}{Z}$

$$y = r \times 2 \ Cos \left[\frac{\frac{2\pi}{\lambda}(vt + x) + \frac{2\pi}{\lambda}(vt - x)}{2} \right]$$

$$Sin \left[\frac{\frac{2\pi}{\lambda}(vt + x) - \frac{2\pi}{\lambda}(vt - x)}{2} \right]$$

$$y = 2r \ Cos \left(\frac{\frac{2\pi}{\lambda}vt + \frac{2\pi}{\cancel{\lambda}}\cancel{x} + \frac{2\pi}{\lambda}vt - \frac{2\pi}{\cancel{\lambda}}\cancel{x}}{2} \right)$$

$$\text{Sin}\left(\frac{\frac{2\pi}{\lambda}vt+\frac{2\pi}{\lambda}x-\frac{2\pi}{\lambda}vt+\frac{2\pi}{\lambda}x}{2}\right)$$

$$y=2r\ \text{Cos}\ \frac{2\pi}{\lambda}\left(\frac{2vt}{2}\right).\ \text{Sin}\ \frac{2\pi}{\lambda}\left(\frac{2x}{2}\right)$$

$$y=2r\ \text{Cos}\ \frac{2\pi}{\lambda}vt.\ \text{Sin}\ \frac{2\pi x}{\lambda}$$

$Now, at\ x=0\ \&\ x=L;\ y=0$

1) $At\ x=0,\ y=0\ is\ satisfied$

2) $At\ x=L,\ y=0$

$$y=2\ r\ \text{Cos}\left(\frac{2\pi}{\lambda}vt\right).Sin\left(\frac{2\pi}{\lambda}\times L\right)$$

$$o=2r\ \text{Cos}\left(\frac{2\pi}{\lambda}vt\right).Sin\left(\frac{2\pi}{\lambda}\times L\right)$$

$$Now, r\neq o, \frac{2\pi}{\lambda}vt\neq o\ \text{so,}$$

$$o=Sin\left(\frac{2\pi L}{\lambda}\right)$$

$$Sin\ n\pi=Sin\ \frac{2\pi 2}{\lambda}$$

$$i.e\ L=n\left(\frac{x}{2}\right)$$

1) Let n = 1 (i.e first harmonic mode or fundamental frequency

$$\lambda=\lambda_1$$

$$L = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

Let v = velocity, γ_1 = frequency of first hamonic Mode

$v = \gamma_1 \lambda_1$

$v = \gamma_1 \times 2L$

$$\frac{v}{2L} = \gamma_1$$

If n = 2 (second harmonic Mode or first overtone)

$$L = \lambda 2$$

$v = velocity, \gamma_2 = ferq4$ of second harmoic Mode

$$v = \gamma_2 \lambda_2$$

$$\gamma_2 = \frac{v}{\lambda_2} = \frac{v}{2}$$

$$\gamma 2 = \frac{v \times 2}{L \times 2} \qquad \left(\because \frac{v}{2L} = \gamma_1\right)$$

$$\gamma_2 = 2\gamma_1$$

i.e frequency of second harmonic Mode is twice the frequency of first harmonic Mode

Similarly, $\gamma n = n\gamma_1$ and frequency of n^{th} harmonic Mode is n times the frequency of first harmonic Mode.

3.Differentiate between the types of vibration in closed and open organ pipes?

Ans.1) In closed pipe, the wavelength of nth mode = $\pi n = \frac{4l}{n}$ where n = odd integer whereas in open pipe, $\lambda n = \frac{2l}{n}$ and n = all integer

2) The fundamental frequency of open pipe is twice that of closed pipe of same length.

3) A closed pipe of length $\frac{2}{2}$ produces the same fundamental frequency as an open pipe of length L.

4) For an open pipe, harmonics are present for all integers and for a closed pipe, harmonics are present for only odd integers hence, open pipe gives richer note.

Put the value of n in equation for frequency

$$f = \frac{1}{2l}\sqrt{\frac{T}{AP}}$$

$$f = \frac{1}{2l}\sqrt{\frac{T/A}{P}}$$

$$\text{Now, Force} = \text{Tension and } \frac{Force}{Area} = \text{Stress}$$

$$f = \frac{1}{2l}\sqrt{\frac{stress}{P}} \rightarrow 2)$$

Now, young's Modulus = y = $\frac{Stress}{Strain}$

Or Stress = y$\times$ Strain

Stress = $y \times \frac{\Delta L}{L}$

Put the value of Stress in equation 2)

$$f = \frac{1}{2l}\sqrt{\frac{y \times \Delta L}{L \times P}}$$

$Put\ y = 9\times10^{10}N/m,\ \Delta L=0.05\times10^{-2}ms$

$L = 1.0m,\ P = 9\times10^{3}\ Kg/m^{3}$

$$f = \frac{1}{2\times1}\sqrt{\frac{9\times10^{9}\times0.05\times10^{-2}}{1\times9\times10^{3}}}$$

Since fundamental frequency of a stretched spring $\frac{\alpha 1}{l}$

$$\frac{f_2}{f_1} = \frac{l_1}{l_2} = \frac{50.1}{49.9}$$

$$f_2 = \frac{50.1}{49.1}\ f_1$$

$Now,\ f_2\text{-}f_1=1\ or\ \frac{50.1}{49.9}f_1 - f_1$

$$1 = \frac{50.1f_1 - 49.9f_1}{49.9}$$

$$1 = \frac{0.2f_1}{49.9}$$

$$1\times49.9 = 0.2f_1$$

$$f_1 = \frac{49.9}{0.2}$$

f_1=249.5HZ

And $f_2 - f_1 = 1$

$f_2 = 1 + f_1$

= 1 + 249.5

f_2 = 250.5 HZ

4.A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Ans. Ultrasonic beep frequency emitted by the bat,v = 40 kHz

Velocity of the bat, v_b = 0.03 v

Where, v = velocity of sound in air

The apparent frequency of the sound striking the wall is given as:

$$v' = \left(\frac{v}{v - v_b}\right)v$$

$$= \left(\frac{v}{v - 0.03v}\right)40$$

$$= \frac{40}{0.97} kHz$$

This frequency is reflected by the stationary wall ($v_s = 0$) toward the bat.

The frequency (v'') of the received sound is given by the relation:

$$v'' = \left(\frac{v}{v + v_s}\right)v'$$

$$= \left(\frac{v + 0.3v}{v}\right) \times \frac{40}{0.97}$$

$$= \frac{1.03 \times 40}{0.97} = 42.47\, kHz$$

5. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1} ? (g= 9.8 $m\ s^{-2}$)

Ans. Height of the tower, *s* = 300 m

Initial velocity of the stone, *u* = 0

Acceleration, *a* = g = 9.8 $m\ s^{-2}$

Speed of sound in air = 340 m/s

The time (t_1) taken by the stone to strike the water in the pond can be calculated using the second equation of motion, as:

$$s = st_1 + \frac{1}{2}gt_1^2$$

$$300 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$$

$$\therefore t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82\,s$$

Time taken by the sound to reach the top of the tower, $t_2 = \frac{300}{340} = 0.88\,s$

Therefore, the time after which the splash is heard, $t = t_1 + t_2$

= 7.82 + 0.88 = 8.7 s

---

6. You have learnt that a travelling wave in one dimension is represented by a function *y* = *f* (*x*, *t*)where *x* and *t* must appear in the combination *x*–*v t* or *x* + *v t*, i.e. *y* = *f* (*x* ± *v t*). Is the converse true? Examine if the following functions for *y* can possibly represent a travelling wave:

(a) $(x - vt)^2$ **(b)** $\log\left[\frac{x+vt}{x_0}\right]$ **(c)** $\frac{1}{(x+vt)}$

Ans. No;

(a) Does not represent a wave

(b) Represents a wave

(c) Does not represent a wave

The converse of the given statement is not true. The essential requirement for a function to represent a travelling wave is that it should remain finite for all values of *x* and *t*.

Explanation:

(a) For $x = 0$ and $t = 0$, the function $(x - vt)^2$ becomes 0.

Hence, for $x = 0$ and $t = 0$, the function represents a point and not a wave.

(b) For $x = 0$ and $t = 0$, the function

$$\log\left(\frac{x+vt}{x_0}\right) = \log 0 = \infty$$

Since the function does not converge to a finite value for $x = 0$ and $t = 0$, it represents a travelling wave.

(c) For $x = 0$ and $t = 0$, the function

$$\frac{1}{x+vt} = \frac{1}{0} = \infty$$

Since the function does not converge to a finite value for $x = 0$ and $t = 0$, it does not represent a travelling wave.

---

7. **For the travelling harmonic wave**

y (x, t) = 2.0 cos 2π (10t – 0.0080x + 0.35)

Where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m, (b) 0.5 m, (c), $\frac{\lambda}{2}$ **(d)** $\frac{3\lambda}{4}$

Ans. Equation for a travelling harmonic wave is given as:

y (x, t) = 2.0 cos 2π (10t –0.0080x + 0.35)

= 2.0 cos (20πt –0.016πx + 0.70 π)

Where,

Propagation constant, k = 0.0160 π

Amplitude, a = 2 cm

Angular frequency, ω = 20 π rad/s

Phase difference is given by the relation:

$$\phi = kx = \frac{2\pi}{\lambda}$$

(a) For x = 4 m = 400 cm

Φ = 0.016 π × 400 = 6.4 π rad

(b) For 0.5 m = 50 cm

Φ = 0.016 π × 50 = 0.8 π rad

(c) For $x = \frac{\lambda}{2}$

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi rad$$

(d) For $x = \frac{3\lambda}{4}$

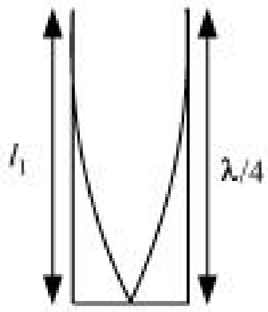
$$\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = 1.5\pi rad$$

---

8. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Ans. Frequency of the turning fork, v = 340 Hz

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure.



Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation:

$$l_1 = \frac{\lambda}{4}$$

Where,

Length of the pipe, $l_1 = 25.5\, cm = 0.255\, m$

$$\therefore \lambda = 4l_1 = 4 \times 0.255\, m = 1.02m$$

The speed of sound is given by the relation:

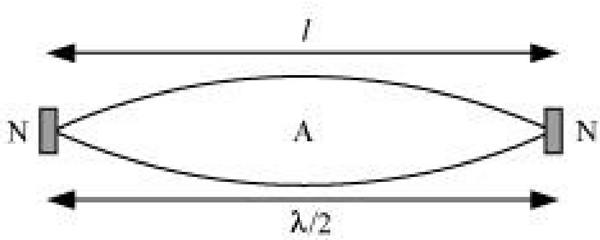
$v = v\lambda = 340 \times 1.02$ = 346.8 m/s

9. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

Ans. Length of the steel rod, l = 100 cm = 1 m

Fundamental frequency of vibration, v = 2.53 kHz = 2.53×10^3 Hz

When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure.



The distance between two successive nodes is $\frac{\lambda}{2}$.

$$\therefore l = \frac{\lambda}{2}$$

$$\lambda = 2l = 2\times 1 = 2m$$

The speed of sound in steel is given by the relation:

v = vλ

= $2.53\times 10^3 \times 2$

= 5.06×10^3 m/s

= 5.06 km/s

(e) A pulse is actually is a combination of waves having different wavelengths. These waves travel in a dispersive medium with different velocities, depending on the nature of the medium. This results in the distortion of the shape of a wave pulse.

10. A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 m s^{-1} , (b) recedes from the platform with a speed of 10 m s-1? (ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 m s-1.

Ans.(i) (a)Frequency of the whistle, v = 400 Hz

Speed of the train, v_T = 10 m/s

Speed of sound, v = 340 m/s

The apparent frequency (v') of the whistle as the train approaches the platform is given by the relation:

$$v' = \left(\frac{v}{v - v_T}\right) v$$

$$= \left(\frac{340}{340-10}\right) \times 400 = 412.12 Hz$$

(b) The apparent frequency (v'') of the whistle as the train recedes from the platform is given by the relation:

$$v'' = \left(\frac{v}{v - v_T}\right) v$$

$$= \left(\frac{340}{340+10}\right) \times 400 = 388.57 Hz$$

(ii) The apparent change in the frequency of sound is caused by the relative motions of the source and the observer. These relative motions produce no effect on the speed of sound. Therefore, the speed of sound in air in both the cases remains the same, i.e., 340 m/s.

---

11. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy

submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m s^{-1}.

Ans. Operating frequency of the SONAR system, v = 40 kHz

Speed of the enemy submarine, v_e = 360 km/h = 100 m/s

Speed of sound in water, v = 1450 m/s

The source is at rest and the observer (enemy submarine) is moving toward it. Hence, the apparent frequency (v') received and reflected by the submarine is given by the relation:

$$v' = \left(\frac{v+v_e}{v}\right)v$$

$$= \left(\frac{1450+100}{1450}\right)\times 40 = 42.76 kHz$$

The frequency (v'') received by the enemy submarine is given by the relation:

$$v'' = \left(\frac{v}{v+v_s}\right)v'$$

Where, $v_s = 100 m/s$

$$\therefore v'' = \left(\frac{1450}{1450-100}\right)\times 42.76 = 45.93 kHz$$

4 Marks Questions

1. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air is 340 m s^{-1}).

Ans. First (Fundamental); No

Length of the pipe, l = 20 cm = 0.2 m

Source frequency = n th normal mode of frequency, v_n = 430 Hz

Speed of sound, v = 340 m/s

In a closed pipe, the nth normal mode of frequency is given by the relation:

$v_n = (2n-1)\frac{v}{4l}$; n is an integer = 0,1,2,3.......

$$430 = (2n-1)\frac{340}{4\times 0.2}$$

$$2n-1 = \frac{430\times 4\times 0.2}{340} = 1.01$$

$$2n = 2.01$$

$$n \sim 1$$

Hence, the first mode of vibration frequency is resonantly excited by the given source.

In a pipe open at both ends, the nth mode of vibration frequency is given by the relation:

$$v_n = \frac{nv}{2l}$$

$$v_n = \frac{2lv_n}{v}$$

$$=\frac{2\times0.2\times430}{340}=0.5$$

Since the number of the mode of vibration (n) has to be an integer, the given source does not produce a resonant vibration in an open pipe.

---

2. Explain why (or how):

(a) In a sound wave, a displacement node is a pressure antinode and vice versa,

(b) Bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",

(c) A violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,

(d) Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) The shape of a pulse gets distorted during propagation in a dispersive medium.

Ans.(a) A node is a point where the amplitude of vibration is the minimum and pressure is the maximum. On the other hand, an antinode is a point where the amplitude of vibration is the maximum and pressure is the minimum.

Therefore, a displacement node is nothing but a pressure antinode, and vice versa.

(b) Bats emit very high-frequency ultrasonic sound waves. These waves get reflected back toward them by obstacles. A bat receives a reflected wave (frequency) and estimates the distance, direction, nature, and size of an obstacle with the help of its brain senses.

(c) The overtones produced by a sitar and a violin, and the strengths of these overtones, are different. Hence, one can distinguish between the notes produced by a sitar and a violin even if they have the same frequency of vibration.

(d) Solids have shear modulus. They can sustain shearing stress. Since fluids do not have any definite shape, they yield to shearing stress. The propagation of a transverse wave is such that it produces shearing stress in a medium. The propagation of such a wave is possible only in solids, and not in gases.

Both solids and fluids have their respective bulk moduli. They can sustain compressive stress. Hence, longitudinal waves can propagate through solids and fluids.

---

3. One end of a long string of linear mass density $8.0\times10^{-3}\ \text{kg m}^{-1}$ is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At *t* = 0, the left end (fork end) of the string *x* = 0 has zero transverse displacement (*y* = 0) and is moving along positive *y*-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement *y* as function of *x* and *t* that describes the wave on the string.

Ans. The equation of a travelling wave propagating along the positive *y*-direction is given by the displacement equation:

$y(x, t) = a \sin(wt - kx)$ … (*i*)

Linear mass density, $\lambda = 8.0\times10^{-3}\ \text{kg m}^{-1}$

Frequency of the tuning fork, v = 256 Hz

Amplitude of the wave, a = 5.0 cm = 0.05 m … (*ii*)

Mass of the pan, m = 90 kg

Tension in the string, $T = mg = 90 \times 9.8 = 882$ N

The velocity of the transverse wave *v*, is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{882}{8.0 \times 10^{-3}}} = 332\, m/s$$

Angular frequency, $\omega = 2\pi v$

$= 2 \times 3.14 \times 256$

$= 1608.5 = 1.6 \times 10^3\, rad/s$ ....(iii)

Wavelength, $\lambda = \frac{v}{v} = \frac{332}{256} m$

$\therefore$ Propagation constant, $k = \frac{2\pi}{\lambda}$

$= \frac{2 \times 3.14}{\frac{332}{256}} = 4.84 m^{-1}$ ....(iv)

Substituting the values from equations (*ii*), (*iii*), and (*iv*) in equation (*i*), we get the displacement equation:

y (*x*, *t*) = 0.05 sin ($1.6 \times 10^3 t - 4.84x$) m

---

4. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (*S*) and longitudinal (*P*) sound waves. Typically the speed of *S* wave is about 4.0 km s-1, and that of *P* wave is 8.0 km s^{-1}. A seismograph records *P* and *S* waves from an earthquake. The first *P* wave arrives 4 min before the first *S* wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Ans. Let v_S and v_P be the velocities of *S* and *P* waves respectively.

Let *L* be the distance between the epicentre and the seismograph.

We have:

$L = v_S t_S$ (*i*)

$L = v_P t_P$ (*ii*)

Where,

t_S *and* t_P are the respective times taken by the *S* and *P* waves to reach the seismograph from the epicentre

It is given that:

v_P = 8 km/s

v_S = 4 km/s

From equations (*i*) and (*ii*), we have:

$v_S t_S = v_P t_P$ 4 t_S = 8 t_P

$t_S = 2t_P$ (*iii*)

It is also given that:

$t_S - t_P$ = 4 min = 240 s

$2t - t_P$ = 240

t_P = 240

And $t_S = 2 \times 240$ = 480 s

From equation (*ii*), we get:

L = 8 $\times$ 240

= 1920 km

Hence, the earthquake occurs at a distance of 1920 km from the seismograph.

5. A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with at a speed of 10 m s^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 m s^{-1}? The speed of sound in still air can be taken as 340 m s^{-1}.

Ans. For the stationary observer: 400 Hz; 0.875 m; 350 m/s

For the running observer: Not exactly identical

For the stationary observer:

Frequency of the sound produced by the whistle, v = 400 Hz

Speed of sound = 340 m/s

Velocity of the wind, v = 10 m/s

As there is no relative motion between the source and the observer, the frequency of the sound heard by the observer will be the same as that produced by the source, i.e., 400 Hz.

The wind is blowing toward the observer. Hence, the effective speed of the sound increases by 10 units, i.e.,

Effective speed of the sound, v_e = 340 + 10 = 350 m/s

The wavelength (λ) of the sound heard by the observer is given by the relation:

$$\lambda = \frac{v_e}{v} - = \frac{350}{400} = 0.875\,m$$

For the running observer:

Velocity of the observer, v_o = 10 m/s

The observer is moving toward the source. As a result of the relative motions of the source and the observer, there is a change in frequency (v').

This is given by the relation:

$$\nu' = \left(\frac{v}{v - v_T} \right) \nu$$

$$= \left(\frac{340+10}{340} \right) \times 400 = 411.76 Hz$$

Since the air is still, the effective speed of sound = 340 + 0 = 340 m/s

The source is at rest. Hence, the wavelength of the sound will not change, i.e., λ remains 0.875 m.

Hence, the given two situations are not exactly identical.

5 Marks Questions

1. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ **to explain why the speed of sound in air**

(a) is independent of pressure,

(b) increases with temperature,

(c) increases with humidity.

Ans. (a) Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \text{ ....(i)}$$

Where,

Density, $\rho = \frac{Mass}{Volume} = \frac{M}{V}$

M= molecular weight of the gas

V= Volume of the gas

Hence, equation(i) reduces to:

$$v = \sqrt{\frac{\gamma PV}{M}} \text{ ......(ii)}$$

Now from the ideal gas equation for $n = 1$:

$PV = RT$

For constant *T*, *PV* = Constant

Since both *M* and Y are constants, *v* = Constant

Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad ......(i)$$

For one mole of an ideal gas, the gas equation can be written as:

PV = *RT*

$$P = \frac{RT}{V} \quad ... (ii)$$

Substituting equation (*ii*) in equation (*i*), we get:

$$v = \sqrt{\frac{\gamma RT}{V\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad . ...(iv)$$

Where,

M = *pv* is a constant

Y and *R* are also constants

We conclude from equation (*iv*) that $v \propto \sqrt{T}$.

Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.

(c) Let v_m and v_d be the speeds of sound in moist air and dry air respectively.

Let ρ_m and ρ_d be the densities of moist air and dry air respectively.

Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Hence, the speed of sound in moist air is:

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad \text{........(i)}$$

And the speed of sound in dry air is:

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \quad \text{...........(ii)}$$

On dividing equations (*i*) and (*ii*), we get:

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma p}{\rho_m} \times \frac{\rho}{\gamma \rho}} = \sqrt{\frac{\rho_d}{\rho_m}}$$

However, the presence of water vapour reduces the density of air, i.e.,

$$\rho_d < \rho_m$$

$$\therefore v_m > v_d$$

Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.

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2. A transverse harmonic wave on a string is described by

$$y(x,t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right) \quad \textbf{........(i)}$$

Where *x* and *y* are in cm and *t* in s. The positive direction of *x* is from left to right.

(a) Is this a travelling wave or a stationary wave?

If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Ans. (a) Yes; Speed = 20 m/s, Direction = Right to left

(b) 3 cm; 5.73 Hz

(c) $\frac{\pi}{4}$

(d) 3.49 m

Explanation:

(a) The equation of a progressive wave travelling from right to left is given by the displacement function:

$y(x, t) = a \sin(\omega t + kx + \theta)$ … (*i*)

The given equation is:

$$y(x,t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right) \text{....(ii)}$$

On comparing both the equations, we find that equation (*ii*) represents a travelling wave, propagating from right to left.

Now, using equations (*i*) and (*ii*), we can write:

ω = 36 rad/s and k = 0.018 m^{-1}

We know that:

$v = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$

Also,

$v = v\lambda$

$$\therefore v = \left(\frac{\omega}{2\pi}\right)\times\left(\frac{2\pi}{k}\right) = \frac{\omega}{k}$$

$$= \frac{36}{0.018} = 2000\, cm/s = 20 m/s$$

Hence, the speed of the given travelling wave is 20 m/s.

(b) Amplitude of the given wave, a = 3 cm

Frequency of the given wave:

$$v = \frac{\omega}{2\pi} = \frac{36}{2\times3.14} = 5.73\, Hz$$

(c) On comparing equations (*i*) and (*ii*), we find that the initial phase angle, $\phi = \frac{\pi}{4}$

(d) The distance between two successive crests or troughs is equal to the wavelength of the wave.

Wavelength is given by the relation:

$$k = \frac{2\pi}{\lambda}$$

$$\therefore k = \frac{2\pi}{\lambda} = \frac{2\times3.14}{0.018} = 348.89 cm = 3.49 m$$

---

3. For the wave described in Exercise 15.8, plot the displacement (*y*) versus (*t*) graphs for *x* = 0, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Ans. All the waves have different phases.

The given transverse harmonic wave is:

$$y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right) \quad .(i)$$

For $x = 0$, the equation reduces to:

$$y(0,t) = 3.0\sin\left(36t + \frac{\pi}{4}\right)$$

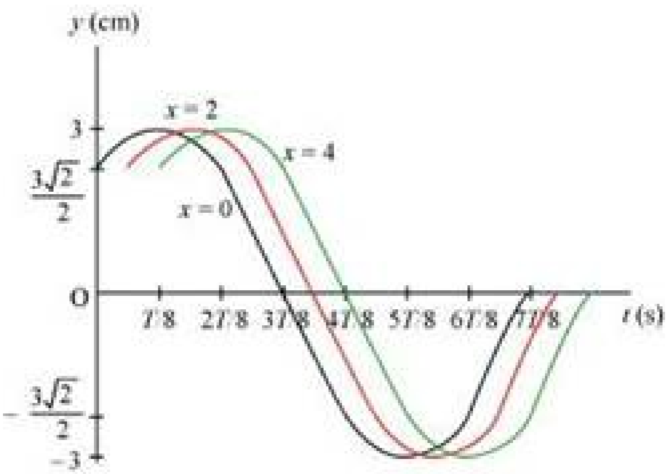
Also, $\omega = \frac{2\pi}{T} = 36\, rad/s^{-1}$

$$\therefore T = \frac{\pi}{8} s$$

Now, plotting y vs. t graphs using the different values of t, as listed in the given table.

t (s)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
y (cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$\frac{-3\sqrt{2}}{2}$	–3	$\frac{-3\sqrt{2}}{2}$	0

For $x = 0$, $x = 2$, and $x = 4$, the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in x. The y-t plots of the three waves are shown in the given figure.



4. The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06 \sin \frac{2}{3} x \cos(120\pi t)$$

Where *x* and *y* are in m and *t* in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

Answer the following:

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

Ans.(a) The general equation representing a stationary wave is given by the displacement function:

$y(x, t) = 2a \sin kx \ \omega \cos °t$

This equation is similar to the given equation:

$$y(x, t) = 0.06 \sin\left(\frac{2}{3} x\right) \cos(120\pi t)$$

Hence, the given function represents a stationary wave.

(b) A wave travelling along the positive *x*-direction is given as:

$$y_1 = a \sin(\omega t - kx)$$

The wave travelling along the negative *x*-direction is given as:

$$y_2 = a \sin(\omega t - kx)$$

The superposition of these two waves yields:

$y = y_1 + y_2 = a\sin(\omega t - kx) - a\sin(\omega t + kx)$

$= a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t) - a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t)$

$= -2a\sin(kx)\cos(\omega t)$

$= -2a\sin\left(\frac{2\pi}{\lambda}x\right)\cos(2\pi\nu t)$ ....(i)

The transverse displacement of the string is given as:

$y(x, t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$ .....(ii)

Comparing equations (*i*) and (*ii*), we have:

$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$

∴Wavelength, h = 3 m

It is given that:

$120\pi = 2\pi$h

Frequency, v = 60 Hz

Wave speed, *v* = vh

= 60 × 3 = 180 m/s

(c) The velocity of a transverse wave travelling in a string is given by the relation:

$v = \sqrt{\frac{T}{\lambda}}$ ....(i)

Where,

Velocity of the transverse wave, *v* = 180 m/s

Mass of the string, m = $3.0\times10^{-2}\ kg$

Length of the string, l = 1.5 m

Mass per unit length of the string, $\lambda = \frac{m}{l}$

$= \frac{3.0}{1.5}\times10^{-2}$

$= 2\times10^{-2}\ kg\ m^{-1}$

Tension in the string = T

From equation (*i*), tension can be obtained as:

$T = v^2\mu$

$= (180)^2\times2\times10^{-2}$

= 648 N

---

5. A travelling harmonic wave on a string is described by

$$y(x,t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

(a) What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.

Ans.(a) The given harmonic wave is:

$$y(x,t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

For x = 1 cm and t = 1s,

$$y(1,1) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

$$= 7.5\sin\left(12.0050 + \frac{\pi}{4}\right)$$

$$= 7.5\sin$$

Where, $\theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.79\,rad$

$$= \frac{180}{3.14} \times 12.79 = 732.81^\circ$$

$$\therefore y = (1,1) = 7.5\sin\left(732.81^\circ\right)$$

$$= 7.5\sin\left(90 \times 8 + 12.81\right) = 7.5\sin 12.81^\circ$$

$$= 7.5 \times 0.2217$$

$$= 1.6629 \approx 1.663\,cm$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt}y(x,t) = \frac{d}{dt}\left[7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)\right]$$

$$7.5 \times 12\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

A t_x =1 cm and t=l s:

$$v = y(1,1) = 90\cos\left(12.005 + \frac{\pi}{4}\right)$$

$$= 90\cos\left(732.81^\circ\right) = 90\cos\left(90\cos \times 8 + 12.81^\circ\right)$$

$= 90\cos\left(12.81^{\circ}\right)$

$= 90 \times 0.975 = 87.75 cm / s$

Now, the equation of a propagating wave is given by:

$y(x,t) = a\sin\left(kx + wt + \phi\right)$

Where,

$k = \frac{2\pi}{\lambda}$

$\therefore k = \frac{2\pi}{\lambda}$

And $\omega = 2\pi v$

$\therefore v = \frac{\omega}{2\pi}$

Speed, $v = v\lambda = \frac{\omega}{k}$

Where,

ω =12 rad/s

K = $0.0050\, m^{-1}$

$\therefore v = \frac{12}{0.0050} = 2400\, cm / s$

Hence, the velocity of the wave oscillation at x = 1 cm and t = 1 s is not equal to the velocity of the wave propagation.

(b) Propagation constant is related to wavelength as:

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{2\pi}{k} = \frac{2\times 3.14}{0.0050}$$

=1253 cm = 12.56 m

Therefore, all the points at distances $n\lambda$ $(n = \pm 1, \pm 2, \ldots \text{and so on})$, i.e. ± 12.56 m, ± 25.12 m, … and so on for *x* = 1 cm, will have the same displacement as the *x* = 1 cm points at *t* = 2 s, 5 s, and 11 s.